Industry dynamics in complex product spaces: An evolutionary model

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ABSTRACT

In this paper we present an evolutionary simulation model of industry dynamics with product innovation and differentiated demand in complex product industries, i.e. industries where products are made of many components, possibly belonging to different technologies, and providing a variety of services to consumers who have heterogeneous preferences. We analyze how the complexity of the product space, the strategies that firms follow to search this space either innovating or imitating, and the differentiation of consumers’ preferences interact to determine the structure and evolutions of the industry.

1. Introduction

From a macro-economic perspective Saviotti have been keen to stress the importance of variety in economic systems in order to promote and sustain growth (Saviotti, 1996; Saviotti and Pyka, 2008). A crucial mechanism of his vision is the necessity of economic sectors to finance the resources required for innovation, which is the core element of growth. Without variety costs-reducing competition would limit the resources available for innovation, depressing further growth and variety, with the system approaching a Malthusian steady state at zero growth. Hence, we expect that variety, represented by new and diversified sectors, ensures positive profits for investments in further innovation, and eventually results as a necessary condition for growth.

The link between variety, innovation and the accompanying financial requirements is however mostly ignored in more traditional evolutionary models of industrial dynamics. Evolutionary models of industrial change, pioneered by Nelson and Winter (1982), have mostly concentrated on process innovation, typically modelled by an increase in labour productivity, in an industry where firms compete by producing a homogeneous product. Though very fruitful, this perspective has mostly overlooked some important empirical facts and theoretical developments that, on the contrary, evolutionary theory should be well equipped to deal with.

First, it is beyond dispute that nowadays a big part of the innovative effort is devoted to product innovation and that generating a continuous stream of product innovations has become a key competitive factor in many industries. Besides, process innovation also often originates by a stream of (product) innovations in capital goods, reinforcing the pressure to consider the economic effects of innovations embodied within products.

Second, in many industries this continuous stream of product innovation goes in parallel with product diversification. Far from competing on homogeneous products, firms use innovation to bring to the market ever new varieties of products and create new market niches. Product
differentiation on the supply side is the counterpart of the differentiation of demand. Buyers are heterogeneous in their needs and preferences and markets are segmented. Thus product innovation is constantly creating sub-markets (Klepper and Thompson, 2006), i.e. transforming industries into systems of weakly competing heterogeneous market segments, with new segments appearing all the time and attracting new potential buyers, and old segments disappearing.

Third, in many industries such products are more and more multi-component and multi-technology. Actually such multiplicity of components and technologies is what enables and feeds diversification. The standard model of one technology, one product, and one market is less and less adequate to describe the reality of many important industries and especially their patterns of innovation. Products are complex bundles of technical components which map into service characteristics (Saviotti and Metcalfe, 1984; Saviotti, 1996). Unlike the genotype–phenotype relation, this mapping of technical components into service characteristics is a key mechanism which drives evolutionary dynamics: selection operates on service characteristics but the unit of selection are the underlying bundles of technical features.

A recent line of enquiry has begun analysing technologies and products as complex systems made of many non-linearly interacting components. Using for instance fitness landscapes models (Kauffman, 1993) it is possible to show that as the extent of interdependencies among components increases, the search space becomes more and more rugged and less and less correlated, that is characterized by many local optima and by large and non-monotonic variations of performance also for small changes in the value of components. In such search spaces usually only local and myopic search is possible because of their combinatory nature and because interdependencies are only partly known, thus global knowledge of the search space is not possible and only a small portion of the search space can actually be explored. However the presence of interdependencies makes local search highly sub-optimal and path dependent as it will only locate the nearest local optimum and be unable to move from there.

However insightful are these results, they have mostly concentrated so far on the properties of search processes of individual organization and have been seldom been extended to the encompass market competition forces as drivers of search (one of the very few notable exceptions being Lenox et al., 2006). In particular, two forces are particular relevant to determine the evolutionary dynamics of firms searching on a complex technological or product space. First, firms are not interested in technological performance per se, but in the profits that may (or may not) derive from it. In turn profits contribute to determine, along with other financial resources, the amount of R&D effort, both innovative and imitative, and therefore the intensity, speed and possibly the scope of search. Second, the landscape on which firms move is not only determined by the technological or product performance surface, but also by demand. Also in this respect technological performance per se does not have much meaning: products have to be sold to potential buyers who indeed value the performance of a product but also its price and the extent to which its characteristics match their own specific taste. If consumers have heterogeneous tastes the technological landscape on which firms move is not only rugged and multi-peaked, but different areas of such landscape may be diversely populated by potential consumers and may therefore present diverse profit opportunities.

In this paper we propose an evolutionary simulation model, or rather a general framework for a breed of possible simulation models, of industry dynamics centred upon the interplay between market competition and product innovation as search in a complex space. The model is meant to be a first step towards a broader understanding of how competition shapes technological innovation, and how innovative patterns affect the competitive environment.

Concerning the model structure, it may be thought as made of two overlapping modules: market competition and technological innovation. The competitive environment is represented by a demand formed by a population of consumers with heterogeneous tastes for product characteristics whose purchasing decisions are represented by a standard utility function comprising products’ quality, prices and individual tastes. The novelty of the model in respect of economic competition lies in the representation of suppliers as endowed with two complementary strategies: innovation/imitation and pricing. The former strategy allows firms to improve the appeal of their products to consumers, while the latter varies prices depending on the relative competitive condition. In particular, contrary to most of the evolutionary models based on mark-up pricing, in our model prices are endogenously determined by each firm in order to exploit technological leadership, or to compensate for technological inferiority, in order to maximize the expected profits.1

The link from competitive conditions to technological innovation are the (realized) profits that determine the amount of resources available for innovation. The reverse link, from innovations to products’ quality affecting consumers’ decisions, depends on the complexity of the technological space, that is central in our model. Products are made of many interdependent components and possess many interdependent characteristics. Firms can innovate by introducing better performing components and/or by introducing new combinations of components. Components can be improved by means of innovation (search for a new product) and imitation (copy the products of more profitable firms). Both innovation and imitation, operated at the level of components, are difficult and uncertain, in the sense that interdependencies among components generate complex trade-offs and there is no guarantee that better components will necessarily increase the overall performance of the product.

1 Note that we are not assuming that our firms can actually maximize profits, but only that they take decisions aiming at that objective. The gap between the purported aim and actual result depends on the uncertainty of future technological innovations and competitor’s actions. Assuming “hard” Knightian uncertainty, expectations in our model suffer from systematic bias, preventing even stochastic ex post profits maximization.
The paper is organized as follows: in Section 2 we briefly review the related literature. In Section 3 we present the core elements of the model. In Section 4 we discuss some insights obtained by exploring the behaviour of the model under several assumptions on the model parameters. Finally in Section 5 we draw the main conclusions.

2. Previous literature

This paper tries and combine two recent and still relatively scarce streams of research: evolutionary models of industry dynamics with product innovation and heterogeneous demand and evolutionary models of technologies and products as complex systems. As to the former, studies have mainly concentrated on heterogeneous consumer preferences as source of product and industry life cycles and as factors that may prevent the emergence of a dominant design. Savio (1996) and Windrum and Birchenn (1998) model the emergences of market niches along the product life cycle as a consequence of the heterogeneous tastes of consumers. Adner and Levinth (2001), Adner (2003), Windrun and Birchenn (2005) and Shy (1996) study the conditions under which a new technology can displace an old one due to network externalities and/or bandwagon effects. Malerba et al. (2007) instead of network externalities and bandwagon effects focus upon the role of pioneer, "experimental" consumers who enable a market niche to get enough momentum for firms serving it to survive and invest in technological progress. Dawid and Reimann (2005) on the contrary considers the drawbacks of competition based upon product diversification that, by pushing firms to introduce new products at a fast rate, may finally hinder product quality.

Turning to the modelling of products as complex system, a recent family of models has used Stuart Kauffman's model of fitness landscapes, the so-called NK model (Kauffman, 1993) and generalizations thereof (Altenberg, 1995; Frenken et al., 1999). These models show that when products and technologies are complex systems made of many non-linearly and non-monotonically interacting elements, local improvements and decentralized search coordinated by market mechanisms are ineffective search strategies. On the other had decomposition of the search space and modular search strategies are necessary in order to make the complexity manageable. Thus the evolution of the system is a matter of a delicate balance between decomposition and integration with multiple equilibria and path dependency characterizing it (Frenken, 2006a; Marengo et al., 2005; Marengo and Dosi, 2006; Auerswald et al., 2000; Ethiraj and Levinth, 2002).

Usually however these papers do not refer to competitive forces and market demand as the basic engine fuelling and directing search in the technological space (notable exceptions being Lenox et al., 2006 and Ciarli et al., 2008): firms search the space implicitly motivated by the quest for better fit technologies and products. Moreover in the fitness landscape models the search space is finite and is therefore inappropriate to model open-ended processes of continuous innovation. In this paper we shall try and fill this gap, suggesting a new variant of the model with an infinite technological space, searched by firms motivated not by technological improvements per se but by the quest for higher profits. Thus firms move on space that is actually the result of the coupling between the technological space and the space constructed by demand, where the latter is determined by a population of heterogeneous consumers. In the following section we describe such a model.

3. A model of competition in complex technological spaces

We model the market for a complex semi-durable product, that is one made of multiple components interacting together to produce an overall performance made of multiple functionalities. Firms compete on the basis of prices, quality of the product and by offering different combinations of characteristics. Moreover they invest profits into R&D activities that may allow them to introduce new and more profitable products into the market and/or imitate the products of better performing competitors. Products are demanded by a population of potential buyers. Potential buyers enter progressively the market, have initially a limited knowledge of the characteristics of the products but they refine it as time passes. After making the first purchase they replace the good at random intervals (as already mentioned the good is durable and has random finite life) by buying the product that, according to their knowledge of its characteristics, maximizes individual utility. The latter is a function of price, quality or performance of the product and its proximity in the space of characteristics to an idiosyncratic "ideal" profile that characterizes each consumer. In the sequel we describe the main features of the model.

3.1. Product space

We model products as systems made of n components \( \{x_1, x_2, \ldots, x_n\} \). Each component can take one out of a set of values \( x_j \in \mathbb{N}^+ \), which are labels for different and progressively "better", in a mere technological sense, types of components (e.g. different CPU types, different wing shapes, different brake cooling systems, etc.).\(^2\) We call \( X \) the set of all the possible products, i.e. of the vectors \( X = [x_1, x_2, \ldots, x_n] \) with \( x_j \in \mathbb{N}^+ \).

The performance (or quality) \( f \) of a product \( x \) is a function of its specific combination of components:

\[
f(x) = \sum_{i=1}^{n} \left( x_i - \sum_{j=1}^{n} (c_{i,j} \cdot |x_i - x_j|) \right) + B \tag{1}
\]

\(^2\) We suppose that there is a clear direction of technological improvement for each component, that is that in some engineering sense we can say that today's CPU is more performing than a CPU of some years ago. This is reflected in our model that firms try to move forward (towards higher values) in the space of components and not backwards. However, because of interdependencies, a higher valued component does not necessarily improve the overall performance of a product if the other components are not co-adapted. Thus at the product level although there is a notional direction for technological improvement, the path may be very rugged and go through deep valleys on lower performance.
where $\epsilon_{i,j} \in [0, 1]$ represents how the contribution of component $i$ to the overall quality is affected by component $j$, and $B$ is a constant.\(^3\)

The overall performance of a product depends both on the values of its components (higher values tend to determine higher performance) but also on their “compatibility” that is tuned by the parameters $\epsilon_{i,j}$. If $\epsilon_{i,j} = 0 \forall i, j$ components are independent and any improvement in any component increases the performance of the product. If, on the contrary, $\epsilon_{i,j} \neq 0 \forall i, j$, better components increase overall performance only insofar as they are compatible with the others. In particular, Eq. (1) assumes that in order to be fully compatible components must have the same value.

This kind of functionalities or characteristics representation of products is both reminiscent of the some models of genotype–phenotype maps in biology and in particular of those based upon generalized NK fitness landscapes (Altenberg, 1995) and of some models of product and industry evolution in the economics of technical change (Saviotti, 1996). For a discussion of the relationships between these two apparently distant lines of research the reader is referred to Frenken (2006).

Moreover, our definition of complexity includes and extends the standard representation used for instance in Kauffman’s NK model of fitness landscape (Kauffman, 1993), where complexity is given by the $K$ parameter that stands for the sheer number of coupled components. In our model we not only may represent the presence of interactions between components by setting $|\epsilon_{i,j}| \neq 0$ when components $x_i$ and $x_j$ are coupled, but also tune the intensity of the interdependency\(^4\) by choosing the value of $|\epsilon_{i,j}| \in [0, 1]$.

As to the extent of such interdependencies, single components may interact with just a few others, or vice versa all $n$ components may interact together. A special and important case is when interactions have a modular or quasi-decomposable structure (Simon, 1969; Baldwin and Clark, 2000), i.e. when the set of components is divided into subsets characterized by strong interactions within each subset and weak or no interactions among subsets. The reader is referred to Frenken et al. (1999); Marengo et al. (2005) and Marengo and Dosi (2005) for a more detailed and formal treatment of these cases and their properties.

All in all, the features of the performance surface describe the difficulty\(^5\) of the innovation process. At one extreme we have the case without interdependencies and with high correlation among the performances of similar products, in which autonomous local (i.e. on single components) improvements can generate a steady stream of innovation. Innovation can be effectively decentralized and innovators can specialize on single components or small modules, whereas coordination is effectively ensured by market selection forces. At the other extreme we have non-monotonic widespread interactions which generate uncorrelated performance surfaces. In this case autonomous local changes are generally ineffective and innovation requires coordinated search on many, possibly all, components together and a deliberate re-designing of the system. Decentralization is highly ineffective in the latter case (see Marengo and Dosi, 2005 for a more detailed and formal development of these arguments).

3.2. Demand

We model two phenomena contributing together to the formation of aggregate demand for the industry: the dynamics of the number of purchases, determined by a process of entry of new consumers and a process of replacement of the installed base with new products, and the dynamics of demand for each product, determined by individual consumers’ choices, in turn based upon price and quality of the products and how well they fit heterogeneous individual tastes on the characteristics.

3.2.1. Consumers’ entry

We model an emergent market that initially contains only a handful of firms and a small set $N_0$ of consumers. Then, new consumers progressively enter the market following a sort of word-of-mouth pattern. Each consumer in $N_0$ has a small number $H$ of acquaintances, and one of them at each period may be introduced to the product and may become a new consumer if buying the product gives positive utility. In turn each of these new consumers has $H − 1$ acquaintances who may then be introduced to the product; and so on until the $H + 1$-th cohort of consumers, who do not have other acquaintances apart from those already in the market.

The result is a S-shaped temporal pattern of the number of consumers in the market. The total dimension of the market depends on $N_0$ and $H$, while the speed of consumer entry depends on the time lag between the arrival of new consumers. We assume that the introduction of new consumers follows a stochastic function governing the time of entry of new consumers. The model imposes 0 probability of introducing a new acquaintance when a consumer just entered the market itself. At each time step the probability is increased by a small amount, until a new entry occurs. At this moment the consumer who “introduced” the new entrant has its probability of introducing a new consumer reset to zero.

Consumers enter the market by purchasing one unit of the good and then make a new purchase at each time step with a probability that increases with the time elapsed since the last purchase. Consequently, at any one time we have only a share of all consumers actually making a purchase, while the complementing share hang on to the product currently owned.

3.2.2. Consumers’ behaviour

When making a purchase, a consumer looks at prices, overall performance and the combination of characteristics of products. We follow the literature on discrete choice

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\(^3\) Since performance values are later used to compute consumers’ utility of consumers we want them to be always non-negative and set $B$ accordingly.

\(^4\) See Valente (2008b) for a discussion on the limits of the NK model and on the properties of possible alternative representations.

\(^5\) This is only one of the many possible sources of difficulty or complexity of technological innovation, the one which stems from the interdependencies between the parts of the technological system and of the underlying knowledge. Other possible sources are not modelled here.
model for products defined in the space of characteristics, and in particular Anderson et al. (1989). Each consumer has an ideal product profile, i.e. her type \( t_i \), with \( \sum_{h=1}^{n} t_{i,h} = 1 \), defined by an “ideal combination” of characteristics the consumer would like to find in the product.

A consumer’s utility depends upon four factors, namely the overall product performance, the distance between the product profile and the consumer’s ideal one, the price and a normally distributed random error. We assume that the elasticities of utility with respect to the first three factors are consumer specific.

The utility of consumer \( i \) buying product \( x \) is given by

\[
U_i(x) = A_i \left( \frac{1}{p_j} \right) w_f^{j,i} d_{i,j} \epsilon(i, t) \tag{2}
\]

where \( f_j \) and \( p_j \) are performance and price of product \( x \), \( d_{i,j} \) is the distance between the product’s profile and consumer’s type \( t_i \), \( \epsilon(i, t) \) is a normally distributed error centred on 1 and whose standard deviation varies with time: \( \epsilon(i, t) \sim \mathcal{N}(1, \sigma(t - t_i)) \). The standard deviation \( \sigma(t - t_i) \) is a decreasing function starting at a given level at the time of entry \( t_i \) of consumer \( i \) in the market, and then decreasing asymptotically to 0. This term represents a learning process by consumers who are assumed to poorly evaluate the products on offer when first entering the market but increase their knowledge of the true product characteristics as they gain more experience. Finally, \( w_f^{j,i} \), \( w_p^{j,i} \) and \( w_\epsilon^{j,i} \) are consumer specific elasticities with respect to performance, price and distance and \( A \) is a constant.

The distance \( d_{i,j} \) of product \( j \) from consumer \( i \)’s ideal profile is computed as

\[
d_{i,j} = \sum_{h=1}^{n} x_{j,h} \cdot t_{i,h} \tag{3}
\]

As already mentioned, the good is durable. Therefore after making a purchase a consumer keeps the product for a random number of iterations. Thus, we have two measures of market penetration for each firm: one computed on the actual sales at any period, and the other based on the products owned by consumers at each moment in time. We call the former market shares and the latter installed base shares.

### 3.3. Firms behaviour

Firms produce only one type of product in the amount demanded by the market and take decisions on prices and R&D investment. Concerning R&D investment decisions, we follow the philosophy of evolutionary models of technical change and industrial dynamics (Nelson and Winter, 1982; Winter, 1984, 1993) and assume that firms take routine decisions by applying rules-of-thumb. In particular we assume that firms invest in R&D a given share of their profits, relating potential innovation to competitive success. As to price decisions we assume instead that firms are more rational than usually assumed by evolutionary models, in particular we make the hypothesis that they aim to maximize profits but are myopically rational in the sense they do not act strategically and do not compute optimal prices at every iteration but only at some intervals.

#### 3.3.1. Price decisions

We assume that the price setting procedure is rational, i.e. based upon deliberate profit maximizing calculations and upon perfect knowledge of demand, but non-strategic, in the sense that is based upon the assumption that the other firms will not respond by modifying their prices. In brief, we assume that a price setting firm computes the highest price at which each individual consumer would buy from the firm itself and then computes the profit maximizing price, assuming constant variable costs.

The routine setting the price for a firm is implemented as follows:

1. Find for each consumer the competitors’ product which would maximize his/her utility;
2. For each consumer, compute the price that the firm should charge for its product in order to be chosen by that consumer, i.e. the price that would make the utility of that consumer higher when buying its own product than the best competitor’s;
3. Rank consumers for descending values of such a price;
4. Identify the profit maximizing price.

This routine also identifies expected profits and expected sales, that, as we will see, will be used to decide whether to implement an innovation.

#### 3.3.2. R&D investment, innovation and imitation

Both innovation and imitation are costly and require R&D investment. Excluding, for the sake of simplicity, external financial sources, our firms invest a share of the profits cumulated since last innovation into R&D in order to search the product space either in new directions (innovation) or trying to move closer to a successful competitor (imitation). Firms with low cumulated profits invest only in imitation, which is less costly and risky, while firms with high cumulated profits invest mainly in innovation, depending on a biased random decision assigning higher probability to innovation than to imitation. The amount of profits, and therefore of R&D investment, determines how many “trials” the firm may possibly perform either in the course of an imitation or innovation routine. The higher the number of trials the larger is the technological space explored by a single round of innovation or imitation.

Both innovation and imitation rely on what may be considered as a “search strategy” in the space of technologies. Given the complexity of such a space, an important variable is the breath of search expressed in the number of components on which search is activates. A “narrow” search strategy concentrates R&D resources on only few (possibly only one) components. Conversely, a “broad” search...

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6 We also assume that firms base their pricing decisions upon the long term potential profit, i.e. on the assumption that all consumers whose utility is maximized by product \( x \) will actually buy this product, though, as we mentioned before, consumers do not immediately switch to their utility maximizing product and before they do products and prices may have further changed.
strategy spreads R&D investment over a larger number (possibly all) of components. Given the same amount of R&D investment, a narrow search will tend to produce relatively larger improvements on few components, while a broad search will tend to attempt comparatively smaller improvements but on many components. The breadth of search is represented by the parameter $1 \leq C \leq n$, that is used to decompose the $n$ dimensional product space into blocks made of $C$ components each.

**Imitation.** We model imitative search in a straightforward way: the imitating firms selects a block containing $C$ components. It then chooses randomly one firm for each of the components in the block, with probabilities proportional to the level of sales (obviously excluding itself, but allowing for replication). It then compare the values of the components in its own product as compared to the values in the firms, and ranks the components giving higher priorities to the components with the wider gap, that is, those with the highest potential of improvements by imitation. Subsequently, the firm determines how many mutation trials are possible given its available R&D investment. Finally, mutations are performed, beginning from components whose value for the imitator’s product is further away from those of target product. For each of the available trials a mutation consists in increasing the level of the component by a fixed amount, and then re-ranking the components in the block as specified above. If the target product is perfectly copied before all possible mutations have been performed the procedure halts.

The values to be imitated are influenced by two assumption, one concerning the degree of imitability, and the second on the time profile of imitation. Concerning the maximum level of imitation, the model includes a parameter determining the degree to which components can be imitated. The value of a component $x_i$ can be imitated up to a maximum of $\tau x_i$, with $\tau \in [0, 1]$. If, for example, $\tau = 0.95$ imitators are only able to reach the 95% of the component’s level for the imitated firm. This parameter is meant to include both imperfections in the observability of the actual characteristics of a technology and limits to the imitability of such characteristics (e.g. because of tacit knowledge, firm specific human capital and skills, etc.).

A further assumption on imitation concerns the temporal dynamics of the imitable values. Each value of a component that can be imitated by others, that is, $\tau x_i$, is used as a limit that imitating firms can observe only after sufficient time is passed since the innovation introducing $x_i$. This assumption ensures that innovating firms enjoy a temporary monopoly on their innovation, which is gradually eroded with time as imitating firms develop the awareness to imitate it. In practice, the model includes a variable defined as $x_{\text{maximm}} = \tau x_i$, that determines the maximum level of that may be imitated. When an imitating firm observes the components of imitated ones, however, the value to imitate is a companion variable $x_{\text{imit}} = x_{\text{imit}} + (1 - \alpha) x_{\text{maximm}}$, where $\alpha \in [0, 1]$ is set very close to 1. Consequently, the imitated values do not change immediately after a potentially imitated firm increases a $x_i$ with an innovation. The level that can be imitated grows with time reaching the maximum imitable level only after a sufficiently long period.

After carrying out updating of the components’ values, the imitating firm computes the optimal price and the expected profit it could earn with the new product, using the same price setting procedure described in the previous section. If the expected profit for the new product is higher than the current level the firm adopts the new imitated product, otherwise keeps offering the current one.

More in detail, the routine for imitation follows the following steps:

1. compute the optimal price, and the corresponding expected profits and expected sales of the current product;
2. choose randomly a block, that is a subset of $C$ components;
3. for each component in the block choose a target firm with probabilities proportional to sales;
4. compute the number of possible mutations given the current R&D investment;
5. being $x_h^i$ the $h$-th component of the imitating firm’s product $i$ and $x_{\text{imit}}^h$ the imitable level for same component of the target firm $j$, compute all distances $(x_{h}^i - x_{\text{imit}}^j)$;
6. increase $x_{h}^i$ by 1, starting from components for which the distance from target firms is higher and update distance $(x_{h}^i - x_{\text{imit}}^j)$; the increment is reduced if the distance is smaller than the unit, so that the imitator can never reach levels above $x_{\text{imit}}^j$;
7. decrease by 1 unit the number of possible mutations;
8. if there are still possible mutations and $x_{h}^i < x_{\text{imit}}^j$ for some $h$ within the chosen block, return to point 6;
9. compute performance, optimal price, expected sales, and expected profits of new product;
10. if the new product has has expected profits or equal profits and higher performance adopt it;
11. else keep the old product.

Two further details are worth stressing. First, for the sake of simplicity we assume that imitation is not limited by patents and other property rights. In a companion paper presenting a slightly simpler model we focus on how different intellectual property rights regimes may act upon industry dynamics, innovation and consumers’ welfare (Marengo et al., 2009). Second, coherently with the findings of the empirical literature on technological change, in our model imitation is both costly and difficult. It is costly because also imitation requires R&D investment, and it is difficult because large gaps can be filled only step by step, and, as already discussed, when products are complex such stepwise imitation strategy does not guarantee to deliver a better product. The larger the gap the more R&D investment is needed to fill it and the more likely that performance will drop while attempting to imitate.

**Innovation.** The routine for innovation is similar to the one for imitation, except that of course there is no target product to be imitated, but mutations are made in random directions starting from the firm’s current product. Also in
this case the number of possible improvements is proportional to R&D investment and the latter is divided among a number of components given by the firm's breath of search parameter $C$.

Differently from the case of imitation, each product generated during an innovation round is a candidate for sale, thus its optimal price and the expected profits are computed. After all possible mutations have been performed, only the most profitable product among the current one and all those generated during the innovation round will actually be marketed. Moreover, we also assume that the size of the steps in innovative search decreases with the technological level already reached, introducing a form of decreasing returns of innovation.

The routine for innovation follows the following steps:

1. compute the optimal price, the corresponding expected profits, and the expected sales of the current product;
2. choose randomly a block, that is a subset of $C$ components;
3. increase the value $x_h$ of the randomly chosen component in the block by $1/x_h$;
4. compute performance, optimal price, expected sales, and expected profits of the new product;
5. decrease by 1 unit the number of possible mutations;
6. if there are still possible mutations to be made, return to point 3;
7. if the best version tested provides higher expected profit adopt it;
8. else do not implement innovation and keep the old product.

3.3.3. Entry and exit

When a firm does not sell any unit of product for a given number of iterations, then it exits the market. At each iteration a new firm may enter with a small probability. New entrants randomly copy components of existing products in the market and price it following the same procedure as all other firms.

4. Simulation results

In this section we present some results produced by testing the model on a few configurations.\(^7\) We discuss below the goals of the simulation exercises we aim to address, and then devote the further paragraphs to discuss a few insights gained by the analysis of the simulation results.

4.1. Simulation goals

The goal of the exercises discussed below is obviously not to assess universal properties of the model, nor to fully test the effects of each and every parameter of the model. These two objectives are not only precluded because they are un-feasible given the complexity of the dynamics involved and the dimension of the parameters' space, but they are also, in our opinion, not worth pursuing. In fact, we are not proposing our model as a universal formal system whose properties are reflected in real-world markets, which would require a thorough analytical or quantitative assessment. Rather, we consider the model a logical system for high level, abstract representation of markets to be used for investigating the logical consequences deriving from the assumptions implemented in the model's equations. A simulation run allows to observe the effects of the assumptions, and its analysis offers the opportunity to reason about the aggregate and dynamic effects generated by a relatively large number of entities through a relatively long virtual time. The simulation exercises are therefore used to answer specific questions related to the elements implemented in the model and concerning the phenomena unfolding during the virtual history of a simulation run. Answering these questions, we claim, enables us to gain insight on the understanding of real-world situations.

The issues that we claim our model can cast light on concern the phenomena generated by the interplay of market dynamics (e.g. market concentration, pricing, profits, etc.) and technological search on complex spaces (how easy it is to get products' quality improvements by local myopic search). The model can be thought as producing two opposing forces. On the one hand we have implemented a positive cycle linking profits to innovation, which favour market concentration. A firm enjoying an initial higher quality product is likely to gain higher profits enabling larger R&D investments, which will reinforce its market leadership. On the other hand, price competition reduces profits for all producers, restricting the scope for innovation because of lower profits, and giving even more competitive relevance to prices. The former cycle will produce a maximally concentrated market where a market leader will sustain a continuous flow of innovation. The latter will instead result in a competitive market where many firms barely survive in a state of continuous price war, resulting in little or no resources to spare for R&D, producing little or no innovation.

In the following paragraphs we will discuss how the balance between the two forces is affected by three conditions: complexity of the technological space, search strategy adopted by firms in order to innovate or imitate, and ease of imitation.

The complexity of the technological space is controlled by setting the number of the interdependency parameters $\epsilon_{i,j} \neq 0$. We will consider two extreme cases: maximum and minimum complexity. The maximum complexity is produced when all coefficients are positive, so that every component affects the contribution of every other component, and the value of the parameter is maximum, that is $\epsilon_{i,j} = 1$, $\forall i,j$. Conversely, minimum complexity is generated by setting all interdependency parameters to 0: $\epsilon_{i,j} = 0$, $\forall i,j$.

The search strategies of firms concern the dimension of the blocks that firms use to divide the search space, that is, the number of components they can vary simultaneously while innovating or mutating, that we referred to as
C. Again, we consider the extreme cases of maximally and minimally modular strategies. The maximum in modularity consists in having $C = 1$, in which an innovating firm operates on a single component at each round of innovation. Conversely, the least modular, or the most integral, strategy consist in always operating on all components, $C = n$.

Finally, the coefficient for the maximum level of imitation $\tau$ is set to either of two levels, 0.95 and 0.99. Obviously, the first case makes imitation less efficient than in the second case, since imitating firms can only reach 95% of the components levels of the imitated products.

The next paragraph describes briefly the basic features of the market dynamics common to all the simulation runs. The remaining paragraphs comment upon the interpretation of the results produced by the different combinations of the parameters.

4.2. Overall dynamics

We first report on some basic variables describing the dynamics of the market. Fig. 1 shows the number of consumers in the market, whose dynamics reflects the entry process discussed in Section 3.2.1.

Sales increase accordingly and generate a typical pattern characterized by an initial large number of firms followed by a shake-out. This pattern is caused by the steep increase of consumers’s entry and by the fact that they initially make mistakes in the evaluation of the products, thus their purchases are dispersed among a large numbers of firms.

As consumers improve their knowledge of the true characteristics of the products the selection pressure on producers becomes more intense. Depending on the parametrization used, the market may generate an oligopolistic core formed by a few firms offering superior products, or produce directly a monopoly where only one firm dominates the market. In both cases we do observe a strong increment of concentration. Fig. 2 shows the number of firms active on the market (upper series) and the inverse Herfindal (dispersion) index (lower series).

The results we will present below are based on the study of simulation runs, analysing individual series and comparing them across different settings. Given the methodological approach used, we base the support of our claims on the logic of the explanations proposed for the phenomena generated in the simulations. The graph reported are meant to provide a visual support to our reasoning, and are meant to be only a presentation tool. Robustness tests are not reported, since the relevant results are, as said, logically deriving from the model’s assumptions, or simply not relevant. For example, for simulations ending up in a monopoly each run provides a different firm as the eventual monopoly, but our claim consists in the existence of monopoly, not in the identification of the monopolist.

For obvious reasons of space we will limit to report for each configuration the time series pattern of overall quality for all firms of each simulation run. These graphs permit also to appreciate, indirectly, other properties of the simulated system. For example, “short” lines spanning only a few time steps refer to firms that enter and exit rapidly, evidently failing to gain sufficient profits to survive longer. A large number of performance series at similar levels indicates a relatively dispersed market, where many firms can be expected to make positive sales. Conversely, a single line far above those of competitors reflect a high concentration, particularly when the technological laggard show a short life, indicating that they make no sales and consequently no profits.

4.3. Complexity and innovation

As first exercise let us consider the effects of technological complexity on the market. We assume imitation to be relatively difficult, setting $\tau = 0.95$, and firms to adopt maximally modular strategies, operating on single components when innovating and imitating, $C = 1$.

In such conditions we expect that more complex spaces, faced by producers with highly modular search strategies, will limit the possibility of technological improvements. As expected, the overall quality levels improves more markedly in the case where the technological space is modular, without interaction among components (see Fig. 3).

The result changes sensibly when we impose a complex technological space, in which the overall performance of a product includes interactions among all the components.

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8 The simulation program used allows interested readers to generate graphs and statistics for every variable in the model.
Fig. 3. Firms’ product performance through time in a simple technological space \( (\epsilon_{ij} = 0, \forall i, j) \), modular search strategies \( (C = 1) \) and difficult imitation \( (\tau = 0.95) \).

Fig. 4 reports the performance of all products under these conditions.

This figure shows that the quality levels for all firms are sensibly lower than the previous modular case. For a relatively long period, after the usual initial period with low concentration, a few firms survive with relatively constant quality levels, not being able to break out of the local peaks reached. Interestingly, we can see that there are two groups of firms, with different quality levels, indicating a segmentation of demand where different groups of consumers consistently choose products with a characteristics’ profile closer to a group of consumers’ tastes. At about the last fifth of the simulation run a “radical” innovation, introduced by a new entrant, breaks the old order, temporarily unifying the two different segments, though quickly new firms reach a still higher quality levels.

All in all, the simulations confirms that a higher complexity makes more difficult to reach high quality levels, a result not surprising given the existing literature on modularity and complexity. However, we can appreciate its implications within a context of market competition. In a simple technological space a modular innovation strategy succeeds in producing more and more quality increments. Such improvements allow the innovative firm to gain extra-profits that finance further innovation, extending the probability of sustained market leadership. Conversely, a complex technological space makes modular innovation far less effective. In this case, price competition is more likely to intensify, reducing profits and consequently the resources available for research investments. Consequently, quality improvements are far less frequent and smaller. In our settings the technological lock-in’s affect incumbents only, since new entrants pick each component’s initial quality from different firms. Consequently, they are allowed to mix and match different profiles. Though most of the entrants will not be able to produce a good quality product, a few lucky ones have the chance to succeed in finding a high quality product by picking the best components from different incumbents. The considerations concerning technological innovation are also reflected on the market conditions. The first case, with low complexity, generates a highly concentrated market, where a single market leader faces only short-lived new entrants, who never reach the dimensions required to challenge the leader’s R&D investments. The second case, with a highly complex technological space, generates instead a more articulated market distribution, with market segmentation reflecting consumers’ tastes. The complexity of the technology and the within-segment competition (depressing prices) prevents incumbents from producing a sustained technological growth.

Complex technologies embody interactions among components that are ignored when innovators adopt a modular search strategy. However, we expect that when firms increase the range of research on more than one component, the innovating efforts should succeed in producing product improvements with higher probability, since the effects of interactions among modules are taken into account. Fig. 5, to be compared with the equivalent Fig. 4 where firms adopt modular search strategies, confirm this hypothesis showing how broad research strategies, encompassing all the modules, allow innovators to successfully deal with the complexity of the technological space in generating higher quality.

Interestingly, however, the complexity of technological space ensures a relatively high degree of dispersion. This is due to the fact that the complex space make the overall growth of performance, though feasible, anyway quite difficult. In these conditions the variety of demand, as represented by consumers’ heterogeneous tastes concerning the relative composition of single components qualities, becomes relatively more relevant. Such phenomenon is responsible for the apparently puzzling result that, though rarely, some firms make innovations resulting in decreasing performance. This is due to the fact that firms do not pursue technological improvements, but profits’ growth. The complexity of the space, in fact, generates cases in

Fig. 4. Firms’ product performance through time in a complex technological space \( (\epsilon_{ij} = 1, \forall i, j) \), modular search strategies \( (C = 1) \) and difficult imitation \( (\tau = 0.95) \).

Fig. 5. Firms’ product performances through time in a complex technological space \( (\epsilon_{ij} = 1, \forall i, j) \), integral search strategies \( (C = 10) \) and difficult imitation \( (\tau = 0.95) \).
which technologically inferior “innovations” (producing a decrease in overall performance) increase the appeal to some segment of consumers, and become consequently attractive to profit-seeking producers.

4.4. Innovation and imitation

The role of imitation with respect to innovation is not univocal. In fact, on the one hand we can expect imitation to hinder innovation because the limited time span of technological superiority is likely to put a strong pressure on prices, and consequently on profits and R&D investment. However, imitation of successful components may increase the number of combinations experimented by producers, therefore increasing the probability of discovering high-quality products.

The following two figures confirm both these hypotheses, showing that the eventual net effect depends on the complexity of the technological space. If the technological space is “simple”, without interaction among components, than the innovation-depressing role of imitation is relatively strong, and we observe a lower level of quality increments in respect of cases in which imitation is more difficult (compare Fig. 6 with the previous Fig. 3).

Conversely, when the technological space is complex, imitation allows for an increasing variety of the supply side, so that what could be a market with no or little innovation, we observe relatively more frequent and diffused quality increments. In a sense, imitation serves as a counter-balance to the myopia of modular search in a complex space, as we deduce by comparing Fig. 7 with the previous Fig. 4, which refers to the same context but with more difficult imitation.

5. Conclusions

This paper is meant to analyze the interactions between two aspects concerning market evolution that, though obviously related in real markets, have been mostly studied separately in the exiting literature: market competition and technological complexity.

Evolutionary theory attributes a central role to innovation as a factor shaping market competition. However it uses a rather simplified representation of technological change, usually modelled as a random draw of a better productivity coefficient, where the size of the improvement depends on the R&D investment. On the other hand, most of the literature on technology as complex system represents technology as a non-homogeneous rugged search space, but assumes that actors freely move on the landscape, being only motivated by the pursue of higher technological efficiency, rather than responding to market forces. Two relevant differences apply when we consider competitive firms engaged in technological competition in a complex landscape.

Firstly, firms are not interested in technological innovation per se, but rather in profits, which are affected by other factors such as prices, the behaviour of competitors and the tastes of consumers. Secondly, firms require resources to fund investments on innovation and imitation. Far from being free, moving in the technological space, is a costly activity that needs to be financed by past profits.

The model proposed in this paper considers a standard, stylised competitive environment represented by two elements. Firstly, consumers are assumed to trade off products’ qualities, prices, and characteristics, creating an economic environment in which firms need to associate an “innovative policy” (how to search for better and better products) with a “pricing policy”, setting the price for their product. We assume that firms try primarily to improve the technological content of their product, and, accordingly, determine the price depending on the demand and competitive conditions.

The pricing strategy adopted by firms consists in setting the price such that expected profits are maximized, assuming firms know the consumers’ preferences and assuming adaptive expectations on competitors’ behaviour.

The model therefore introduces a link between the possibility of innovation and the capacity of funding such activity: though an innovation may be technically within reach, lack of funding may prevent the firm to actually adopt the innovation. Since R&D is funded by profits, the necessity to fight a price war may reduce the profits to the level in which innovation cannot be pursued for economic reasons, even though it is, in theory, easy to access from a technological point of view.

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Footnote: 9 Note that in our model we do not include expectations, so that imitation plays no role in influencing innovative behaviour in terms of expected profits.
The results reported show that considering technological and economic aspects together provides a richer and more coherent picture of the dynamics of innovation. We showed that overall innovation rates and market concentration are influenced by technological and competitive conditions, providing a more general interpretative framework for some standard results. Moreover, we are also able to shed light on the role of imitation, which, on the one hand, determines the competitive pressure felt by innovators, and, on the other hand, stimulates innovation by diffusing novelty.

The model presented in this paper has been exploited only for a few exploratory exercises, but it may easily offer the opportunity for both a deeper analysis of its properties and for further extensions. For example, it is possible to study in greater detail the demographic properties of the population of firms, analysing the effects of complexity on the shape and timing of the shake-out and the subsequent patterns. Also, the analysis of the successful firms may provide insights on the most effective strategies. Finally, it would be interesting to study the effects other parameters in the model, such as the frequency of purchase by the buyers, which affect the timing by which an innovation spreads in the market, or the speed by which innovations may be imitated by competitors.

Concerning possible extensions to the basic version of the model, an interesting development would concern the endogenisation of search strategies. Rather than imposing exogenously a given search strategy, firms may be endowed with a routine selecting upon a set of search strategies, adopting the one that appears as most effective. It would be interesting to see whether firms change their strategies in respect of the different phases of the market maturity and/or of their competitive conditions. Furthermore, it is a common feature of modern markets to develop products endowed with ever increasing functionalities, often quite distant from the primary purpose of the original product. Such type of "innovations" are actually an important means to develop new products, blurring the distinction between incremental and radical innovations. The model may be modified to investigate the possibility to extend the number of components, besides improving the existing ones, and observing how different hypothesis on complexity and demand affect the evolution of a market, its possible segmentation into increasingly distinct sub-markets and, eventually, in new markets for radically different products.

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